Energy Flow from Open to Closed Strings in a Toy Model of Rolling Tachyon

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We study the toy model of interacting open and closed string tachyons which demonstrates some interesting properties of the unstable D-brane decay scenario. We compute a stress tensor of the system and study the energy and pressure dynamics. We show that the total energy of the system is conserved. We separate the stress tensor into two parts corresponding to open and closed strings and study the energy flow from open to closed strings. The two vacua of the system could be interpreted as corresponding to the unstable D-brane background and the open string tachyon vacuum. We study how for spatially homogenous solution interpolating between these two vacua the total energy of the open string dissipates to the closed string.

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I. INTRODUCTION

Physical process of the unstable D-brane decay has recently attracted a lot of attention [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. This process has been investigated in frameworks of boundary CFT [1, 8] and within open string field theory (OSFT) [2, 3, 7, 10]. To work within OSFT a level-truncated scheme has been usually employed. Dynamical equations in this approach are rather interesting from mathematical point of view since they contain an infinite number of derivatives. These equations could be represented in the integral form that corresponds to a nonlocal interaction found in string theories. For detailed analysis of such types of equations see for example [2, 5, 6, 7, 16], for some rigorous mathematical results in this field see [14]. Despite of these mathematical difficulties several interesting results where obtained. In particular special time dependent solutions of classical equations of motion where constructed in p-adic string theory [20, 21, 22] in papers [2, 16] and in SSFT in papers [6, 7]. These solutions describe the tachyon field which starts from the unstable vacuum of the system with non-zero velocity and tends to the stable one reaching it at infinite time. The D-brane's tension at the unstable vacuum is expected to be at its maximum and vanishing while moving to the stable one.

An interesting point in the current development of the unstable D-brane decay is the disagreement between the conformal field theory considerations and the results obtained in the level-truncated open cubic string field theory approximations. This gave rise to an investigation of open-closed interacting string models where it was proposed that one has to take into account the fact that the D-brane's energy dissipates to the closed string while reaching the resulting stable vacuum.

In this paper we consider a model with two interacting tachyon fields recently proposed in [15]. The model could be seen as a simplification of level-truncated Open-Closed String Field Theory (OCSFT) where several terms are omitted even on the first nontrivial level. Although this model could be considered only as a simplified model of OCSFT it has a time dependent solution interpolating between two vacua of the system. This solution starts from the first vacuum interpreted as an unstable D-brane background and tends to the second one where the D-brane disappears [15]. Here we show that for this solution the conserved energy dissipates from the open string tachyon to the closed one. This coincides with the expected behavior of the D-brane's energy.

The paper is organized as follows. In Section II we describe the model and study its dependance on the parameters. Then in Section III we compute a stress tensor of the model and separate the terms corresponding to open and closed tachyons. Finally in Section IV we study energy and pressure dynamics. First, we provide a direct prove of the energy convergence. Then using the separation of terms in the stress tensor we study energy flow from open to closed strings. We show how the energy of open string dissipates to closed one. At last we show a picture of the pressure dynamics. The pressure is negative and vanishes at infinite time.

II. THE TOY MODEL FOR OPEN AND CLOSED STRING TACHYONS

We study a toy model of open-closed string tachyons recently proposed in [15]. In the units with $\alpha' = 1$ the

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corresponding action takes the form

$$S = \int d^D x \left[\frac{1}{2}\phi\Box\phi + \frac{1}{2}\phi^2 + \frac{1}{2}\psi\Box\psi + 2\psi^2 - \frac{1}{3}\tilde{\phi}^3 + c_2\tilde{\phi}\tilde{\psi} - \tilde{\phi}^2\tilde{\psi} \right], \quad (1)$$

where ϕ and ψ are interpreted as open and closed string tachyon fields respectively [15], and one defines

$$\tilde{\phi}(x) \equiv e^{k\Box}\phi(x), \quad (2a)$$

$$\tilde{\psi}(x) \equiv e^{m\Box}\psi(x), \quad (2b)$$

$k = m = \log 2$ with the differential operator $e^{a\Box}$ is understood as a series expansion

$$e^{a\Box} = \sum_{n=0}^{\infty} \frac{a^n \Box^n}{n!}, \quad (3)$$

(about an appearance of nonlocal terms in SFT see reviews [17, 18, 19]) and c_2 is a constant which is discussed below. The space-time is flat with the metric $\eta^{\mu\nu} = \mathrm{diag}(-1, 1, \ldots, 1)$, so that the D'Alamber operator takes the form $\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2$. The equations of motion corresponding to the action (1) are the following

$$\Box\phi + \phi - e^{k\Box}\tilde{\phi}^2 + c_2 e^{k\Box}\tilde{\psi} - 2e^{k\Box}(\tilde{\phi}\tilde{\psi}) = 0 \quad (4a)$$

$$\Box\psi + 4\psi + c_2 e^{m\Box}\tilde{\phi} - e^{m\Box}\tilde{\phi}^2 = 0 \quad (4b)$$

Let us consider spatially homogeneous configurations. In this case the equations of motion (4a)-(4b) take the following form

$$(-\partial^2 + 1)e^{2k\partial^2}\tilde{\phi} - \tilde{\phi}^2 + c_2\tilde{\psi} - 2\tilde{\phi}\tilde{\psi} = 0 \quad (5a)$$

$$(-\partial^2 + 4)e^{2m\partial^2}\tilde{\psi} + c_2\tilde{\phi} - \tilde{\phi}^2 = 0, \quad (5b)$$

where the time derivative is denoted by $\partial = \frac{d}{dt}$ and $\tilde{\phi} = \tilde{\phi}(t)$, $\tilde{\psi} = \tilde{\psi}(t)$.

Equations of motion (5a)-(5b) contain a pseudo-differential operator of the form $e^{a\partial^2}$, it is defined as

$$e^{a\partial^2}\varphi(t) = \int \mathcal{K}_a(t-\tau)\varphi(\tau)d\tau, \quad (6)$$

where $\mathcal{K}_a$ is a Gaussian kernel given by

$$\mathcal{K}_a(x) = \frac{1}{\sqrt{4\pi a}} e^{-\frac{x^2}{4a}} \quad (7)$$

The integral operator (6) is well defined for $a > 0$, see [2, 6, 14, 16] for detailed analysis of equations with such operators.

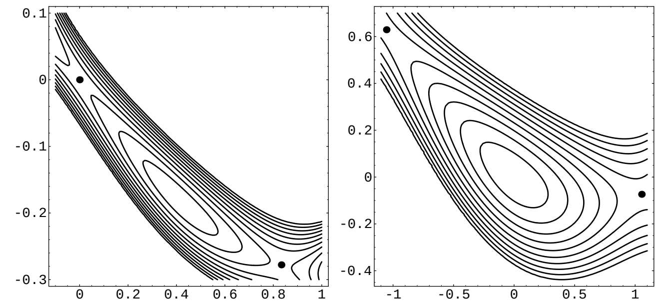

FIG. 1: Equipotential lines of the effective mechanical potential (12). For the case $c_2 = \frac{13}{6}$ (left) there are two vacua with the same energy: $(\tilde{\phi} = 0, \tilde{\psi} = 0)$ and $(\tilde{\phi} = \frac{5}{6}, \tilde{\psi} = -\frac{5}{18})$, for the case $c_2 = \frac{4}{3}$ (right) there are again two distinct vacua with the same energy: $(\tilde{\phi} = \pm\frac{\sqrt{10}}{3}, \tilde{\psi} = \frac{5\mp 2\sqrt{10}}{18})$. On the figures the corresponding vacua are marked with thick dots.

The equations of motion have tree vacua solutions which could be easily found by the following procedure. First we solves the equation (4b) for constant ϕ, ψ with respect to ψ. This gives $\psi = (\phi^2 - c_2\phi)/4$. Now substituting it to (4a) and using the fact that the fields are constant we get an equation $\phi(2\phi^2 + (4-3c_2)\phi + c_2^2 - 4) = 0$ which has three solutions

$$\phi_0 = 0, \quad \phi_{1,2} = \frac{1}{4}(3c_2 - 4 \pm \sqrt{c_2^2 - 24c_2 + 48})$$

these are the vacua of the system in terms of c_2.

The potential of the system has the form

$$V = \frac{1}{8}\phi^4 + \left(-\frac{1}{4}c_2 + \frac{1}{3}\right)\phi^3 + \left(\frac{1}{8}c_2^2 - \frac{1}{2}\right)\phi^2 \quad (8)$$

Now we want to fix a constant c_2 in such a way that there are two distinct vacua with the same energy, i.e. we solve equations $V(\phi_i) = V(\phi_j)$, $i, j = 0, 1, 2$ with respect to c_2 and check that for this value of c_2 $\phi_i \neq \phi_j$. The first step gives us the following values of c_2

$$c_2 = \pm 2, \quad c_2 = \frac{13}{6}, \quad c_2 = \frac{4}{3}, \quad c_2 = 4(3 \pm \sqrt{6})$$

Although only for the values $c_2 = \frac{13}{6}$ and $c_2 = \frac{4}{3}$ there are two *distinct* vacua with the same energy.

In [15] a space homogeneous configuration was constructed numerically for the case $c_2 = \frac{13}{6}$ using the following iterative procedure

$$\begin{aligned} \tilde{\phi}_{n+1} &= \frac{1}{c_2}(\tilde{\phi}_n^2 - \mathcal{P}_4\tilde{\psi}_n), \\ \tilde{\psi}_{n+1} &= \frac{1}{c_2}(-\mathcal{P}_1\tilde{\phi}_n + \tilde{\phi}_n^2 + 2\tilde{\phi}_n\tilde{\psi}_n), \end{aligned} \quad (9)$$

where $n = 1, 2, \ldots$ and the initial configuration is taken as $\phi_0(t) = \frac{5}{6}\theta(t)$, $\psi_0(t) = -\frac{5}{18}\theta(t)$, where $\theta(t)$ is a step function being 1 for $t > 0$ and 0 otherwise. The operator $\mathcal{P}_r = (-\partial^2 + r)e^{2k\partial^2}$ could be presented in the fully

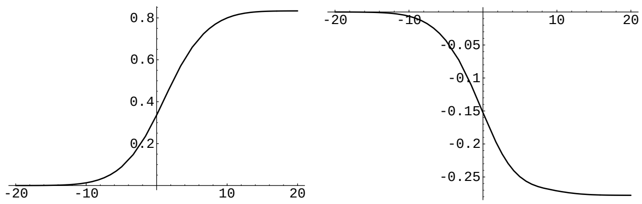

FIG. 2: Spatially homogeneous solutions for the case $c_2 = \frac{13}{6}$ constructed using the iterative procedure (9): $\phi(t)$ (left) and $\psi(t)$ (right).

integral form by differentiating the kernel (7)

$$(\mathcal{P}_r\varphi)(t) = \int (-\frac{d^2}{dt^2} + r)K_{2k}(t-\tau)\varphi(\tau)d\tau \tag{10}$$

The results of the iterative procedure (9) are presented on Fig.2.

In order to support this numerical results let us study the equations (5a)-(5b) neglecting high order derivatives. This reduces the original equations to the following system (see (2a)-(2b))

$$(2k-1)\partial^2\tilde{\phi} + \tilde{\phi} - \tilde{\phi}^2 + c_2\tilde{\psi} - 2\tilde{\phi}\tilde{\psi} = 0 \tag{11a}$$

$$(8m-1)\partial^2\tilde{\psi} + 4\tilde{\psi} + c_2\tilde{\phi} - \tilde{\phi}^2 = 0 \tag{11b}$$

Equations (11a)-(11b) could be seen as describing a mechanical system with the following potential

$$V_{\text{eff}}(\tilde{\phi},\tilde{\psi}) = \frac{1}{2}\tilde{\phi}^2 + 2\tilde{\psi}^2 - \frac{1}{3}\tilde{\phi}^3 + c_2\tilde{\phi}\tilde{\psi} - \tilde{\phi}^2\tilde{\psi}. \tag{12}$$

We will call potential obtained in such a way an effective mechanical potential. Note that

$$V_{\text{eff}}\big(\tilde{\phi}, \frac{1}{4}(\tilde{\phi}^2 - c_2\tilde{\phi})\big) = -V(\tilde{\phi}) \tag{13}$$

this reflects the fact that interacting terms contain second order derivatives and thus contribute to the kinetic term. We have this effect of potential-flipping only if $k > \frac{1}{2}$ and $m > \frac{1}{8}$ (we have this in our model since $k = m = 2$), i.e. the effective "masses" are positive.

The described construction of an effective mechanical potential provide good intuition about the existence of rolling-type solutions. Indeed the shape of V_{eff} (Fig.1) points (at least for the case $c_2 = \frac{13}{6}$) to the existence of kink-type solutions for the reduced system (11a)-(11b) and this usually a good sign for the original nonlocal system to have a rolling-type solution [6, 7]. The direct proof of existence of rolling-type solutions for our system might be rather nontrivial. Indeed the properties of the integral kernel (10) does not allow us to use the same approach as was used in [14].

Although the iterative procedure (9) demonstrates a good numerical convergence for $c_2 = \frac{13}{6}$ it rapidly diverges for $c_2 = \frac{4}{3}$ where the initial functions where taken to be "stepping" between the corresponding vacua

$$\phi_0 = \pm\frac{\sqrt{10}}{3}, \quad \psi_0 = \frac{5 \mp 2\sqrt{10}}{18}, \tag{14}$$

It is left unclear whether kink-type solution exists even for the reduced system (11a)-(11b) for this value of c_2. Our numerical investigations show that the arch at the bottom of the potential for this case (see Fig.1, part right) prevents the trajectory started from one endpoint (14) to reach the second.

Below we will refer to the spatially homogeneous solution for the case $c_2 = \frac{13}{6}$ displayed on Fig.2 [15].

III. STRESS TENSOR

Let us compute a stress tensor for the model and present it in the form with the separated terms corresponding to open and closed string.

We use the definition of a stress tensor from general relativity

$$T_{\alpha\beta} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\alpha\beta}} \tag{15}$$

We make the action (1) invariant under space-time transformations by including metric in the action

$$S = \int d^Dx\sqrt{-g}\left[\frac{1}{2}\phi\Box\phi + \frac{1}{2}\phi^2 + \frac{1}{2}\psi\Box\psi + 2\psi^2 - \frac{1}{3}\tilde{\phi}^3 + c_2\tilde{\phi}\tilde{\psi} - \tilde{\phi}^2\tilde{\psi}\right] \tag{16}$$

where the D'Alamber operator $\Box$ is covariant

$$\Box = \frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu$$

While performing a variation of the action (16) according to (15) we will have to deal with the following expressions

$$\int d^Dy\ f(y)\frac{\delta\Box_y}{\delta g_{\alpha\beta}(x)}h(y) \tag{17}$$

and

$$\int d^Dy\ f(y)\frac{\delta\tilde{\chi}(y)}{\delta g_{\alpha\beta}(x)}, \quad \tilde{\chi}(y) \equiv e^{n\Box}\chi(y), \tag{18}$$

here and below the index y denotes derivation with respect to y.

Let us now consider these expressions in more detail [5, 7]. Using the following equality

$$\frac{\delta\sqrt{-g(y)}}{\delta g^{\alpha\beta}(x)} = -\frac{1}{2}\sqrt{-g(y)}g_{\alpha\beta}(y)\delta(x-y)$$

we compute

$$\frac{\delta\Box_y}{\delta g^{\alpha\beta}(x)} = \frac{g_{\alpha\beta}}{2}\delta(x-y)\Box_y + \frac{1}{\sqrt{-g(y)}}\partial_{y_\mu}\Big[\frac{-\sqrt{-g(y)}}{2}\delta(x-y)g_{\alpha\beta}(y)g^{\mu\nu}(y) + \sqrt{-g(y)}\delta^{\mu\nu}_{\alpha\beta}\delta(x-y)\Big]\partial_{y_\nu} \quad (19)$$

Now for the expression (17) we obtain

$$\int d^Dy\ f(y)\frac{\delta\Box_y}{\delta g_{\alpha\beta}(x)}h(y) = \int d^Dy\ f(y) \times\Big[\frac{g_{\alpha\beta}}{2}\delta(x-y)\Box_y + \frac{1}{\sqrt{-g}}\partial_{y_\mu}\Big(\frac{-\sqrt{-g}}{2}\delta(x-y) \times g_{\alpha\beta}g^{\mu\nu} + \sqrt{-g}\delta^{\mu\nu}_{\alpha\beta}\delta(x-y)\Big)\partial_{y_\nu}\Big]h(y) \quad (20)$$

Now let us consider the expression (18). First let us rewrite it in the following form

$$\int d^Dy\ f(y)\frac{\delta\tilde{\chi}(y)}{\delta g_{\alpha\beta}(x)} = \int d^Dy\ f(y)\frac{\delta e^{n\Box}}{\delta g_{\alpha\beta}(x)}\chi(y)$$

Using (19) and the equality [5]

$$\frac{\delta\hat{A}}{\delta g^{\alpha\beta}(x)} = \int_0^1 d\rho e^{\rho\hat{A}}(\frac{\delta\hat{A}}{\delta g^{\alpha\beta}(x)})e^{(1-\rho)\hat{A}},$$

we obtain

$$\int d^Dy\ f(y)\frac{\delta\tilde{\chi}(y)}{\delta g_{\alpha\beta}(x)} = \frac{g_{\alpha\beta}}{2}\Big[n\int_0^1 d\rho(e^{n\rho\Box}f)(\Box e^{n(1-\rho)\Box}\chi) + +n\int_0^1 d\rho(\partial_\mu e^{n\rho\Box}f)(\partial^\mu e^{n(1-\rho)\Box}\chi)\Big] - n\int_0^1 d\rho(\partial_\alpha e^{n\rho\Box}f)(\partial_\beta e^{n(1-\rho)\Box}\chi) \quad (21)$$

Now using (20) and (21) we are able to compute the stress tensor of the system

$$T_{\alpha\beta} = T^\phi_{\alpha\beta} + T^\psi_{\alpha\beta}, \quad (22)$$

where the part corresponding to the open string tachyon is

$$T^\phi_{\alpha\beta} = -g_{\alpha\beta}\Big[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\phi^2 - \frac{1}{3}\tilde{\phi}^3\Big] - \partial_\alpha\phi\partial_\beta\phi - g_{\alpha\beta}\Big[k\int_0^1 d\rho(e^{-(2-\rho)k\Box}(\Box+1)\tilde{\phi})(\Box e^{-k\rho\Box}\tilde{\phi}) + k\int_0^1 d\rho(\partial_\mu e^{-(2-\rho)k\Box}(\Box+1)\tilde{\phi})(\partial^\mu e^{-k\rho\Box}\tilde{\phi})\Big] + 2k\int_0^1 d\rho(\partial_\alpha e^{-(2-\rho)k\Box}(\Box+1)\tilde{\phi})(\partial_\beta e^{-k\rho\Box}\tilde{\phi}),$$

and the part corresponding to the closed string tachyon is given by

$$T^\psi_{\alpha\beta} = -g_{\alpha\beta}\Big[-\frac{1}{2}\partial_\mu\psi\partial^\mu\psi + 2\psi^2 - \tilde{\psi}e^{-2m\Box}(\Box+4)\tilde{\psi}\Big] - \partial_\alpha\psi\partial_\beta\psi - g_{\alpha\beta}\Big[m\int_0^1 d\rho(e^{-(2-\rho)m\Box}(\Box+4)\tilde{\psi})(\Box e^{-m\rho\Box}\tilde{\psi}) + m\int_0^1 d\rho(\partial_\mu e^{-(2-\rho)m\Box}(\Box+4)\tilde{\psi})(\partial^\mu e^{-m\rho\Box}\tilde{\psi})\Big] + 2m\int_0^1 d\rho(\partial_\alpha e^{-(2-\rho)m\Box}(\Box+4)\tilde{\psi})(\partial_\beta e^{-m\rho\Box}\tilde{\psi}),$$

where we used the equations of motion. We see that the terms of the stress tensor corresponding to open and closed strings are separated. This allows us to study the energy flow from open to closed strings.

IV. ENERGY AND PRESSURE DYNAMICS

The energy of the system is defined in terms of the stress tensor (22)

$$E = T^0_0 \quad (23)$$

As it was shown in the previous section it is possible to separate the terms of the stress tensor corresponding to open and closed strings. Thus the energy of the system (23) could also be written as a sum of energies of open and closed strings.

For spatially homogeneous configurations the energy takes the form

$$E(t) = E_\phi(t) + E_\psi(t), \quad (24)$$

where the term corresponding to open string is given by

$$E_\phi(t) = -\frac{1}{2}\phi^2 + \frac{1}{2}(\partial\phi)^2 + \frac{1}{3}\tilde{\phi}^3 + k\int_0^1 d\rho(e^{(2-\rho)k\partial^2}\{-\partial^2+1\}\tilde{\phi})\overleftrightarrow{\partial}(e^{k\rho\partial^2}\partial\tilde{\phi}),$$

and the term corresponding to closed string is

$$E_\psi(t) = -2\psi^2 + \frac{1}{2}(\partial\psi)^2 + \tilde{\psi}\Big[(-\partial^2+4)e^{2m\partial^2}\tilde{\psi}\Big] + m\int_0^1 d\rho(e^{(2-\rho)m\partial^2}\{-\partial^2+4\}\tilde{\psi})\overleftrightarrow{\partial}(e^{m\rho\partial^2}\partial\tilde{\psi}),$$

here and below $A\overleftrightarrow{\partial}B = A\partial B - B\partial A$.

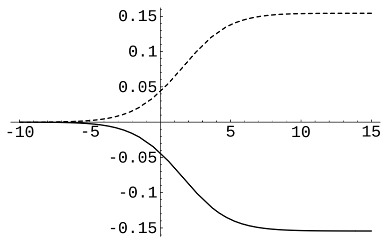

FIG. 3: The the energy of closed string E_ψ (dashed line) and the energy of open string E_ϕ (solid line).

One can immediately see that this energy conserves

$$
\begin{aligned}
\frac{dE(t)}{dt} &= -\phi\partial\phi - 4\psi\partial\psi + \tilde{\phi}^2\partial\tilde{\phi} + \partial\phi\partial^2\phi + \partial\psi\partial^2\psi \\
&+ \partial\tilde{\psi}\left[(-\partial^2+4)e^{2m\partial^2}\tilde{\psi}\right] + \tilde{\psi}\left[(-\partial^2+4)e^{2m\partial^2}\partial\tilde{\psi}\right] \\
&+ k\int_0^1 d\rho(e^{(2-\rho)k\partial^2}\{-\partial^2+1\}\tilde{\phi})\overleftrightarrow{\partial^2}(e^{k\rho\partial^2}\partial\tilde{\phi}) \\
&+ m\int_0^1 d\rho(e^{(2-\rho)m\partial^2}\{-\partial^2+4\}\tilde{\psi})\overleftrightarrow{\partial^2}(e^{m\rho\partial^2}\partial\tilde{\psi}),
\end{aligned}
$$

using the identity [7]

$$
m\int_0^1 d\rho(e^{m\rho\partial^2}\varphi)\overleftrightarrow{\partial^2}(e^{m(1-\rho)\partial^2}\phi) = \varphi\overleftrightarrow{e^{m\partial^2}}\phi, \qquad (25)
$$

we get

$$
\begin{aligned}
\frac{dE(t)}{dt} &= -\phi\partial\phi - 4\psi\partial\psi + \tilde{\phi}^2\partial\tilde{\phi} + \partial\phi\partial^2\phi + \partial\psi\partial^2\psi \\
&+ \partial\tilde{\psi}\left[(-\partial^2+4)e^{2m\partial^2}\tilde{\psi}\right] + \tilde{\psi}\left[(-\partial^2+4)e^{2m\partial^2}\partial\tilde{\psi}\right] \\
&- (\partial\tilde{\phi})\overleftrightarrow{e^{k\partial^2}}(\{-\partial^2+1\}\phi) - (\partial\tilde{\psi})\overleftrightarrow{e^{m\partial^2}}(\{-\partial^2+4\}\psi)
\end{aligned}
$$

or simplifying

$$
= \tilde{\phi}^2\partial\tilde{\phi} - \partial\tilde{\phi}\left[\{-\partial^2+1\}e^{k\partial^2}\phi\right] + \tilde{\psi}\left[\{-\partial^2+4\}e^{m\partial^2}\partial\psi\right]
$$

now using the equation of motion (4b) we get

$$
\begin{aligned}
&= \tilde{\phi}^2\partial\tilde{\phi} - \partial\tilde{\phi}\left[\{-\partial^2+1\}e^{k\partial^2}\phi\right] + \tilde{\psi}\partial(\tilde{\phi}^2 - c_2\tilde{\phi}) \\
&= \partial\tilde{\phi}\left[-(-\partial^2+1)e^{k\partial^2}\phi + \tilde{\phi}^2 - c_2\tilde{\psi} + 2\tilde{\phi}\tilde{\psi}\right] \equiv 0
\end{aligned}
$$

We see that the time derivative of the energy is zero on the equations of motion and thus the total energy is conserved. We also see that on infinities the energy is zero and thus $E(t) = 0$ or $E_\phi(t) = -E_\psi(t)$ at all times. The energy flow from open to closed string for the solution described in the section II is presented on Fig.3. We see that the energy contained in the open string tachyon dissipates to the closed string tachyon that could be interpreted as a transformation of the unstable D-brane's energy to the energy of closed string [15].

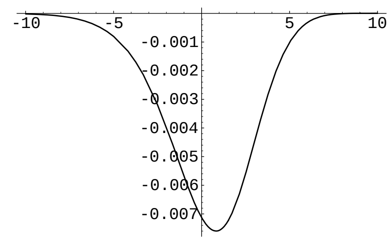

FIG. 4: The pressure of the system (27) is negative and vanishes on infinities.

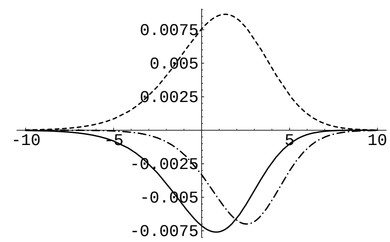

FIG. 5: The positive term $(\partial\phi)^2 + (\partial\psi)^2$ of the pressure (dashed line), the negative terms (dashed-dotted line) and the total pressure (solid line). We see that there is a nontrivial compensation which leads to negative total pressure.

Let us now investigate the pressure of the system [2, 5, 7]. The pressure is defined in terms of the stress tensor as

$$
p_i = -T_i^i \quad \text{(no summation in } i\text{)}. \qquad (26)
$$

Since we consider spatially homogeneous configurations we will omit the vector index i. The pressure dynamics is given by

$$
\begin{aligned}
p(t) &= -E(t) + (\partial\phi)^2 + (\partial\psi)^2 \\
&- 2k\int_0^1 d\rho(e^{(2-\rho)k\partial^2}\{-\partial^2+1\}\partial\tilde{\phi})(e^{k\rho\partial^2}\partial\tilde{\phi}) \\
&- 2m\int_0^1 d\rho(e^{(2-\rho)m\partial^2}\{-\partial^2+4\}\partial\tilde{\psi})(e^{m\rho\partial^2}\partial\tilde{\psi}) \qquad (27)
\end{aligned}
$$

The pressure dynamics is presented on Fig.4. Since the total energy is zero and the fields are constant at infinities we see from (27) that the pressure vanishes as $|t| \to \infty$. The pressure contains two positive terms with only second-order derivatives of ϕ and ψ and two negative terms with high-order derivatives of $\tilde{\phi}$ and $\tilde{\psi}$. These terms are presented separately on Fig.5. We see that there is a nontrivial compensation of the positive terms making the total pressure being negative.

Let us note that the form (27) is rather nontrivial from computational point of view. In particular we where able to present the last two terms in such a way that the first exponent is well defined in terms of the integral operator (6) for all $0 \leqslant \rho \leqslant 1$ while there is no such possibility for the second exponent in both terms – the Gaussian kernel diverges at $\rho = 0$. Although for ρ near to 0 we where able to use the series expansion (3) without loss of precision.

V. CONCLUSION

In this paper a simplified model of the unstable D-brane decay in the open-closed string field theory was studied. We have computed a stress tensor of the system and presented it as a sum of two terms corresponding to open and closed strings. We have obtained the energy of the system and proved its convergence. The separation of energies of open and closed strings allowed us to study the energy flow from open to closed strings. We have considered a time-dependant solution interpolating between two vacua of the system for the case $c_2 = \frac{13}{6}$. We have showed that the energy of open string dissipates to the closed string. The question of existence of such interpolating solution for the case $c_2 = \frac{4}{3}$ is left open. The next important step would be to generalize these results to the full level-truncated OCSFT and study the D-brane's decay there.

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